# Study of the Structure of Photonic Crystal Fiber with High Negative Dispersion Coefficient 

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#### Abstract

: The optical fiber communication has been second topics only to robot study for today. In the process of the Dense Wave-Length Division Multiplexing (DWDM) study, the problem of the dispersion compensate for the traditional optical fiber is a difficult problem to be solved for long distance transport information. In order to solve this problem, it is half work and times time by experimental study on photonic crystals with high negative dispersion structure. The advanced COMSOL Multiphysics many physical fields coupling calculation software is preferred. The research methods are that the structural parameters are adjusted for the traditional hexagonal photonic crystal fiber and the negative dispersion coefficient is obtained as far as possible large. Then the structure that the several layers with same spacing is designed and the structure is with ultra-high negative dispersion coefficient. The result shows that it is several ten times of the domestic level and it is 1.1 times of the international level. It is times work and half time by COMSOL Multiphysics many physical fields coupling calculation software in Modeling, mesh subdivision, calculation and analysis. The result is the theory basis for DWDM.


Keywords: Information optics; Photonic crystal fiber, Negative dispersion coefficient, Dispersion compensation

## 1 Introduction

transferred in a fiber if the declining coefficient was less than $20 \mathrm{~dB} / \mathrm{km}$ in 1964. The development of the optical fiber technology has five stages. Now the optical fiber is main carrying information for the communication. With the life level of the people is rising and the information quantity is more and more. The communication technology is toward to dense Wave-Length Division Multiplexing (DWDM). The dispersion problem is the obstruction for the DWDM ${ }^{[1]}$.

There is the dispersion for the optical fiber of G. 652 with 1550 nm and the dispersion makes the disturbance between the several signs. The dispersion compensation is benefit to restrain the optical pulse widen ${ }^{[3-5]}$ so the design that is to obtain the largest negative dispersion coefficient is very important. The dual-core microstructure fiber designed by Huttunen, et al. in 2005 and the negative dispersion coefficient was -59000ps/(nm $\cdot \mathrm{km})^{[6]}$. Xu , et al. designed hexagonal dual-core photonic crystal fiber and the ultra-level dispersion area with 480 nm near $1550 \mathrm{~nm}{ }^{[7]}$.

## 2 The theoretical basis of optical transfer in photonic crystal fiber

Galerkin finite element method is suit for calculation and showing the results of the photonic crystal fiber with different cross sections. Galerkin finite element method is effect one for analyzing the transfer characteristics to quartz or other material.

The magnetic field component that obeyed the vector wave equation with Maxwell's equations is as follow.

$$
\begin{equation*}
\nabla \times\left[\frac{1}{\varepsilon_{r}} \nabla \times \boldsymbol{H}\right]-k_{0}^{2} \boldsymbol{H}=0 \tag{1}
\end{equation*}
$$

Here, $\boldsymbol{H}$ is the vector of magnetic field. The symbol $k_{0}$ is wave number in vacuum. The symbol $\varepsilon_{r}$ is relative dielectric constant. The expression of the magnetic field component along to $z$ direction is

$$
\begin{equation*}
H(x, y, z, t)=\left[H_{x}, H_{y}, H_{z}\right]^{T}(x, y) \exp [\mathrm{i}(\omega t-\beta z)] \tag{2}
\end{equation*}
$$

Here $\beta=k_{0} n_{\text {eff }}$ is transfer parameter and $n_{\text {eff }}$ is effect refractive index.
For the non-magnetic matter it is $\nabla \times \boldsymbol{H}=0$. The vector wave equation only in transverse magnetic field is

$$
\left[\begin{array}{c}
\frac{\partial}{\partial y}\left[\frac{1}{n_{z z}^{2}}\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right)\right]  \tag{3}\\
-\frac{\partial}{\partial x}\left[\frac{1}{n_{z z}^{2}}\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right)\right]
\end{array}\right]-\left[\begin{array}{l}
\frac{1}{n_{y y}^{2}} \frac{\partial}{\partial x}\left(\frac{\partial H_{x}}{\partial x}+\frac{\partial H_{y}}{\partial y}\right) \\
\frac{1}{n_{x x}^{2}} \frac{\partial}{\partial y}\left(\frac{\partial H_{x}}{\partial x}+\frac{\partial H_{y}}{\partial y}\right)
\end{array}\right]+k_{0}^{2} n_{\mathrm{eff}}^{2}\left[\begin{array}{c}
\frac{1}{n_{y y}^{2}} H_{x} \\
\frac{1}{n_{x x}^{2}} H_{y}
\end{array}\right]=k_{0}^{2}\left[\begin{array}{c}
H_{x} \\
H_{y}
\end{array}\right]
$$

Here, $n_{x x}$ is the refractive index in $x$ direction, $n_{y y}$ is the refractive index in $y$ direction, and $n_{z z}$ is the refractive index in $z$ direction.
The variation equation derivation from the Galerkin finite element method is

$$
\begin{align*}
& \iint_{\Omega}\left\{\nabla_{t}\left[\begin{array}{l}
\frac{1}{n_{z z}^{2}} \omega_{y}\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right) \\
\frac{1}{n_{z z}^{2}} \omega_{x}\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right)
\end{array}\right]+\nabla_{t}\left[\begin{array}{l}
\frac{1}{n_{y y}^{2}} \omega_{x}\left(\frac{\partial H_{x}}{\partial x}-\frac{\partial H_{y}}{\partial y}\right) \\
\frac{1}{n_{x x}^{2}} \omega_{y}\left(\frac{\partial H_{x}}{\partial x}-\frac{\partial H_{y}}{\partial y}\right)
\end{array}\right] d \mathrm{dxdy}\right. \\
& +\iint_{\Omega}\left\{\frac{1}{n_{z z}^{2}}\left(\frac{\partial \omega_{y}}{\partial x}-\frac{\partial \omega_{x}}{\partial y}\right)\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right)\right\} \mathrm{d} x \mathrm{~d} y+\iint_{\Omega}\left\{k_{0}^{2} n_{\text {eff }}^{2}\left(\frac{\omega_{x} H_{x}}{n_{y y}^{2}}+\frac{\omega_{y} H_{y}}{n_{x x}^{2}}\right)\right\} \mathrm{d} x \mathrm{~d} y  \tag{4}\\
& +\iint_{\Omega}\left\{\left[\frac{\partial}{\partial x}\left(\frac{\omega_{x}}{n_{y y}^{2}}\right)+\frac{\partial}{\partial y}\left(\frac{\omega_{y}}{n_{x x}^{2}}\right)\right]\left(\frac{\partial H_{x}}{\partial x}+\frac{\partial H_{y}}{\partial y}\right)-k_{0}^{2}\left(\omega_{x} H_{x}+\omega_{y} H_{y}\right)\right\} d x \mathrm{~d} y=0
\end{align*}
$$

Here, $\Omega$ is the calculating area, $\Omega_{e}$ is triangle unit from the $\Omega$. The symbol $\Gamma_{e}$ is outer edge unit and $\Gamma_{\text {int }_{e}}$ is internal edge unit area. The symbol $\nabla_{t}=\boldsymbol{i} \frac{\partial}{\partial x}+\boldsymbol{j} \frac{\partial}{\partial y}$, and $\omega=\left[\omega_{x}, \omega_{y}\right]^{T}$ is weighted function. Expression (4) could be written as

$$
\begin{align*}
& \sum_{B_{e}}\left\{-\int_{\Gamma_{e}} \frac{\omega_{y}}{n_{z z}^{2}}\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right) \mathrm{d} y-\int_{\Gamma_{e}} \frac{\omega_{x}}{n_{z z}^{2}}\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right) \mathrm{d} x\right\} \\
& -\sum_{B_{e}}\left\{-\int_{\Gamma_{e}} \frac{\omega_{x}}{n_{y y}^{2}}\left(\frac{\partial H_{x}}{\partial x}-\frac{\partial H_{y}}{\partial y}\right) \mathrm{d} y-\int_{\Gamma_{e}} \frac{\omega_{x}}{n_{x x}^{2}}\left(\frac{\partial H_{x}}{\partial x}-\frac{\partial H_{y}}{\partial y}\right) \mathrm{d} x\right\} \\
& +\sum_{\text {int }}^{e}\left\{-\int_{\Gamma_{\text {inete }}} \Delta_{x}\left(\frac{1}{n_{y y}^{2}}\right) \omega_{x}\left(\frac{\partial H_{x}}{\partial x}-\frac{\partial H_{y}}{\partial y}\right) \mathrm{d} y-\int_{\Gamma_{\text {inex }}} \Delta_{y}\left(\frac{1}{n_{x x}^{2}}\right) \omega_{y}\left(\frac{\partial H_{x}}{\partial x}+\frac{\partial H_{y}}{\partial y}\right) \mathrm{d} x\right\}  \tag{5}\\
& +\sum_{\text {intere }} \iint_{\Omega_{e}} \frac{1}{n_{z z}^{2}}\left\{\left(\frac{\partial \omega_{y}}{\partial x}-\frac{\partial \omega_{x}}{\partial y}\right)\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right)+\left[\frac{\partial}{\partial_{x}}\left(\frac{\omega_{x}}{n_{y y}^{2}}\right)+\frac{\partial}{\partial_{y}}\left(\frac{\omega_{y}}{n_{x x}^{2}}\right)\right]\left(\frac{\partial H_{x}}{\partial x}+\frac{\partial H_{y}}{\partial y}\right)\right\} \mathrm{d} x \mathrm{~d} y \\
& +\sum_{\text {in } y} \iint_{\Omega_{e}}\left[k_{0}^{2} k_{\text {eff }}^{2}\left(\frac{\omega_{x} H_{x}}{n_{y y}^{2}}+\frac{\omega_{y} H_{y}}{n_{x x}^{2}}\right)-k_{0}^{2}\left(\omega_{x} H_{x}+\omega_{y} H_{y}\right)\right] \mathrm{d} x \mathrm{~d} y=0
\end{align*}
$$

Here, $B_{e}$ is triangle unit divided with grids, int $_{e}$ is the internal edge unit, $\mathrm{int}_{+}$is the outside of the internal edge, int_ is the inside of the internal edge.

$$
\begin{align*}
& \Delta_{x}\left(\frac{1}{n_{y y}^{2}}\right)=\left(\frac{1}{n_{y y}^{2}}\right)_{x=x_{\mathrm{x}+0}}-\left(\frac{1}{n_{y y}^{2}}\right)_{x=x_{i n t}}  \tag{6}\\
& \Delta_{x}\left(\frac{1}{n_{x x}^{2}}\right)=\left(\frac{1}{n_{x x}^{2}}\right)_{y=y_{i n+t}}-\left(\frac{1}{n_{x x}^{2}}\right)_{y=y_{i x+}} \tag{7}
\end{align*}
$$

The dispersion characteristics of the photonic crystal fiber are relationship with the effect
refractive index. The refractive index is calculated by the Galerkin finite element method and the dispersion characteristics could be analyzed in theory.

## 3 The design, modeling, analysis, and calculation

The idea of the design, modeling, analysis, and calculation for photonic crystal fiber is based on three points. The first one is that the refractive index ratio with clad and core is from 1.48:1.46 to 1.46:1, from common fiber to photonic crystal fiber. The second one is that the air holes are arranged in rule around the axis. The third one is that the transferred energy focused on a layer. The designed structure is as figure 1 . The center of the photonic crystal fiber is a defect and it is not an air hole. The air holes are arranged asymmetry around the center. The air holes in the first layer are named as $11,12,13, \ldots$. The air holes in the second layer are named as 21, 22, $23, \ldots$. The air holes in the third layer are named as $31,32,33, \ldots$. The air holes in the fourth layer are named as $41,42,43, \ldots$. The air holes in the fifth layer are named as $51,52,53, \ldots$. The air holes in the sixth layer are named as $61,62,63, \ldots$.


Figure 1 The cross section of the photonic crystal fiber

In figure 1, the coordination of the center is designed as $(0,0)$, the coordination axis is toward to left and upper.

The coordination values of the air holes in first layer are P11 (1,0); P12(0.5, 0.866025); P13(-0.5, $0.866025)$; P14(-1,0).
The coordination values of the air holes in second layer are P21(2,0); P22(1.732051,1); P23(1, 1.732051); P24(0, 2); P25(-1, 1.732051); P26(1.732051, 1); P27(-2,0).

The coordination values of the air holes in third layer are P31(3,0); P32(2.85317, 0.92705); P33(2.427052, 1.763354); P34(1.763354, 2.427052); P35(0.92705, 2.85317); P36(0, 3); P37(1,0); P38(-0.927051, 2.85317); P39(-1.763354, 2.4270520); P3A(-2.427052, 1.763354); P3B(-3,0).
The coordination values of the air holes in fourth layer are P41(4,0); P42(3.912591, 0.831646); P43(3.654182, 1.626945); P44(3.2360692, 2.3511393); P45(2.676525, 2.972577); P46(2,3.4640998); P47(1.236072, 3.80422475); P48(0.418119, 3.978087); P49(-0.418119, 3.978087); P4A(-1.236072, 3.80422475); P4B(-2,3.4640998); P4C(-2.676525, 2.972577); P4D(-3.2360692, 2.3511393); P4E(-3.654182, 1.626945); P4F(-3.912591, 0.831646); P4G(-4,0).
The coordination values of the air holes in fifth layer are P51(5,0); P52(4.924039, 0.86824); P53(4.698464, 1.710099) ; P54(4.330128, 2.5); P55(3.830224, 3.213936) ; P56(3.213936, 3.830224); P57(2.5, 4.330128) ; P58(1.710099, 4.698464); P59(0.86824, 4.924039) ; P5A(0, 5); P5B(-0.86824, 4.924039) ; P5C(-1.710099, 4.698464); P5D(-2.5, 4.330128) ; P5E(-3.213936, 3.830224); P5F(-3.830224, 3.213936) ; P5G(-4.330128, 2.5); P5H(-4.698464, 1.710099) ; P5I(-4.924039, 0.86824); P5J(-5,0).
The coordination values of the air holes in sixth layer are P61(6,0); P62(5.92613, 0.938606); P63(5.70634, 1.8541) ; P64(5.3460402, 2.72394087); P65(4.854104, 3.526709) ; P66(4.2426435, 4.2426435); P67(3.5267154, 4.854099159) ; P68( 2.723948, 5.346037); P69(1.8541, 5.70634) ; P6A(0.938606, 5.92613); P6B(0, 5) ; P6C(-0.938606, 5.92613); P6D(-1.8541, 5.70634) ; P6E(-2.723948, 5.346037); P6F(-3.5267154, 4.854099159) ; P6G(-4.2426435, 4.2426435); P6H(-4.854104, 3.526709) ; P6I(-5.3460402, 2.72394087); P6J(-5.70634, 1.8541) ; P6K(-5.92613, $0.938606)$; P6L(-6,0).

The software system of COMSOL Multiphysics is used for calculation after the photonic crystal fiber was designed. The spacing between the neighbor layers $d_{0}=1.500 \mu \mathrm{~m}$, the diameter of the air hole $d$ is between $1.12 \mu \mathrm{~m} \sim 1.16 \mu \mathrm{~m}$ and its step length is $0.02 \mu \mathrm{~m}$. The transferred wavelength is between $1.500 \mu \mathrm{~m}^{\sim} 1.600 \mu \mathrm{~m}$ and its step length is $0.001 \mu \mathrm{~m}$. There are 101 effect refractive indexes in steady state the transferred wavelength between $1.500 \mu \mathrm{~m}^{\sim} 1.600 \mu \mathrm{~m}$ and
its step length is $0.001 \mu \mathrm{~m}$ and the diameter of the air hole is $1.12 \mu \mathrm{~m} .1 .366893,1.366810$, 1.366728, 1.366645, 1.366563, 1.366480, 1.366398, 1.366315, 1.366233, 1.366150, 1.366067, 1.365985, 1.365902, 1.365819, 1.365737, 1.365654, 1.365571, 1.365489, 1.365406, 1.365323, $1.365240,1.365158,1.365075,1.364992,1.364909,1.364826,1.364744,1.364661,1.364578$, $1.364495,1.364412,1.364329,1.364246,1.364163,1.364081,1.363998,1.363915,1.363832$, 1.363749, 1.363666, 1.363583, 1.363500, 1.363417, 1.363334, 1.363251, 1.363168, 1.363085, 1.363002, 1.362919, 1.362836, 1.362753, 1.362672, 1.362610, 1.362574, 1.362539, 1.362505, 1.362470, 1.362435, 1.362401, 1.362366, 1.362331, 1.362297, 1.362262, 1.362227, 1.362193, 1.362158, 1.362124, 1.362089, 1.362054, 1.362020, 1.361985, 1.361950, 1.361916, 1.361881, 1.361847, 1.361812, 1.361777, 1.361743, 1.361708, 1.361674, 1.361639, 1.361605, 1.361570, $1.361535,1.361501,1.361466,1.361432,1.361397,1.361363,1.361328,1.361293,1.361259$, 1.361224, 1.361190, 1.361155, 1.361121, 1.361086, 1.361052, 1.361017, 1.360983, 1.360948 . The succeed example is that the angles for the first layer air holes to the center are 60 degree. There are six air holes in the first layer. There are 12 air holes in the second layer and the angles for the second layer air holes to the center are 30 degree. There are 20 air holes in the third layer and the angles for the third layer air holes to the center are 18 degree. There are 30 air holes in the fourth layer and the angles for the fourth layer air holes to the center are 12 degree. There are 36 air holes in the fifth layer and the angles for the fifth layer air holes to the center are 10 degree. There are 40 air holes in the sixth layer and the angles for the sixth layer air holes to the center are 9 degree. The spacing from the first layer to center is $d_{0}$. The spacing from the second layer to center is $2 d_{0}$. The spacing from the third layer to center is $3 d_{0}$. The spacing from the fourth layer to center is $4 d_{0}$. The spacing from the fifth layer to center is $5 d_{0}$. The spacing from the sixth layer to center is $6 d_{0}$.
In figure 2, the curve is enlarged for the wavelength scope of $1.545 \mu \mathrm{~m}^{\sim} 1.566 \mu \mathrm{~m}$ and the fitting cubic curve is

$$
\begin{equation*}
n_{\text {eff }}=-134.87 \lambda^{3}+631.04 \lambda^{2}-984.2 \lambda+513.05 \tag{8}
\end{equation*}
$$

The dispersion coefficient is

$$
\begin{equation*}
D=-\frac{\lambda}{c} \frac{\mathrm{~d}^{2}}{\mathrm{~d} \lambda^{2}} n_{\mathrm{eff}}=-\frac{\lambda}{c}(-134.87 \times 6 \lambda+631.04 \times 2) \tag{9}
\end{equation*}
$$

The dispersion coefficient $D=-65207.4 \mathrm{ps} /(\mathrm{nm} \cdot \mathrm{km})$ is obtained for the wavelength being $1.550 \mu \mathrm{~m}$.


Figure 2 The fitting cubic curve of the effect refractive index

## 4 Conclusions

The software system of COMSOL Multiphysics is used for modeling, analysis, and calculation after the multilayer circle asymmetry photonic crystal fiber being designed. The negative dispersion coefficient is to $-65207.4 \mathrm{ps} /(\mathrm{nm} \cdot \mathrm{km})$ at transfer wavelength being 1550 nm . The result is 40 times than that of the reference [8] and is 1.1 times than that of the reference [6]. The 1 meter of the designed structure is suit for the 3260 meters compensation the dispersion of the single optical fiber G. 652.

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