Based on two-dimensional wavelet decomposition time-varying System Identification

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Abstract: In practice, we often encounter non-linear time-varying systems. It is difficult to identificate and model them. In this paper, linear operator deduced wavelet-based time-varying systems described in the modeling process, and gives its identification algorithm.

Key words: Two-dimensional wavelet; Time-varying systems; Scale function; Filter

1 Introduction

The model parameters are changing, so the difficulty of identification is input and output for each set of observation data in time-varying system. The time-varying system identification techniques usually can be divided into two categories. The first identification uses regression recognition technology, such as RPEM method, RLS method ， RIV method and so forth. It can track the slow changes; however, if the parameter changes rapidly, the estimated variance can be substantial. The second type is starting with a variety of orthogonal sequence in time-varying parameters, and then uses least squares estimation. In such method, the choice of basis functions impacts directly on identification results good or bad. The common ones are Lagrange polynomials, Fourier series, discrete flat ball sequence and B-spline and so on. The approach to a variety of orthogonal sequence in time-varying parameter would increase the number of peacekeeping operations significantly.

In this paper, linear operator deduces wavelet-based time-varying systems described in the modeling process. First, the time-varying two-dimensional wavelet coefficients start the time-varying system identification into the identification of time-invariant coefficients.

Second, it uses a cost function to select the best basis function, which can greatly reduce the amount of coefficient, and obtains brief models. The model can describe a relatively simple way of the characteristics of system's and nonlinear time-varying.

2 Definitions and Lemma

(1)Dimensional Tensor Product Multi-resolution Analysis (2DMRA)

Suppose there are two MRA in $L^2\left(R^2\right)$, which respectively are $\left\{V_j^1,\varphi^1\right\}$ and $\left\{V_j^2,\varphi^2\right\}$ corresponding to two wavelet denoted ψ^1 and ψ^2,and

$V_j^1=\left\{\varphi^1(2^j x_1-k_1),k_1\in z\right\}$,$V_j^2=\left\{\varphi^2(2^j x_2-k_2),k_2\in z\right\}$,

$\psi^1\left(x_1\right)=\sum_{k_1} p_{k_1}^1\varphi^1\left(2x_1-k_1\right)$, $\psi^2\left(x_2\right)=\sum_{k_2} p_{k_2}^2\varphi^2\left(2x_2-k_2\right)$.

Define sub-space as

$$F_j=V_j^1\otimes V_j^2=\left\{\varphi^1(2^j x_1-k_1)\varphi^2(2^j x_2-k_2),\left(k_1,k_2\right)\in Z^2\right\}\subset L^2\left(R^2\right),$$

then, the sub-space sequence $\left\{F_j\right\}$ satisfies:

Monotonicity: $F_{j-1}\subset F_j \quad j\in z$;

Approximation: $\bigcap_{j\in z}F_j=\{0\}$, $\overline{\bigcup_{j\in z}F_j}=L^2\left(R^2\right)$;

Scalability: $f\left(x_1,x_2\right)\in F_j\Leftrightarrow f\left(2^{-j}x_1,2^{2-j}x_2\right)\in F_0,\forall j\in z$;

Translation invariance:

$f\left(x_1,x_2\right)\in F_0\Rightarrow f\left(x_1-k_1,x_2-k_2\right)\in F_0,\left(k_1,k_2\right)\in z^2$;

The existence of Riesz basis: The Riesz basis of F_0 consists of the function system $\left\{\varphi^1(2^j x_1-k_1)\varphi^2(2^j x_2-k_2)\right\}_{k_1,k_2\in z^2}$.

And because $V_{j+1}^1=V_j^1\otimes W_j^1$, $V_{j+1}^2=V_j^2\otimes W_j^2$, including $W_j^1=\left\{\psi^1\left(2^j x_1-k_1\right),k_1\in z\right\}$, $W_j^2=\left\{\psi^2\left(2^j x_2-k_2\right),k_2\in z\right\}$.

So ,

$$F_1=V_1^1\otimes V_1^2=F_0\oplus\left(V_0^1\otimes W_0^2\right)\oplus\left(W_0^1\otimes V_0^2\right)\oplus\left(W_0^1\otimes W_0^2\right).$$

(2)Two-dimensional Wavelet Function and Scale Relations

According to the nature of the tensor product, we can see:

$\left\{\psi^1\left(x_1-K_1\right)\psi^2\left(x_2-K_2\right)\right\}$ is $W_0^1 \otimes W_0^2$ Riesz basis;

$\left\{\psi^1\left(x_1-K_1\right)\varphi^2\left(x_2-K_2\right)\right\}$ is $W_0^1 \otimes W_0^2$ Riesz basis;

$\left\{\varphi^1\left(x_1-K_1\right)\psi^2\left(x_2-K_2\right)\right\}$ is $V_0^1 \otimes W_0^2$ Riesz basis.

It can be seen that there are $\psi^{(1)}=\varphi^1\otimes\psi^2$, $\psi^{(2)}=\psi^1\otimes\varphi^2$, $\psi^{(3)}=\psi^1\otimes\psi^2$ in $L^2\left(R^2\right)$, which are the wavelet corresponding to $\varphi\left(x_1,x_2\right)$.

Suppose $\varphi\left(x,y\right)$ is the Two-dimensional scaling function, which satisfies the following two-scale relations:

$$\varphi\left(x,y\right)=\sum_{m,n} p_{m,n}\varphi\ \left(2x-n\right)\varphi\ \left(2y-n\right).$$

$\{p_{m,n}\}$ is a sequence, whose frequency-domain is expressed as $\varphi(2\xi,2\eta)=P(\xi,\eta)\varphi(\xi,\eta)$ .If $\varphi\left(x,y\right)$ is orthonormal scaling function, we can get:

$$<\varphi\left(x,y\right),\varphi\left(x-m,y-n\right)>=\delta_{0,m}\delta_{0,n} \qquad (1)$$

3 Linear Model Described

A time-varying linear system can be described as

$$y\left(t\right)=\int K\left(t,\tau\right)u\left(\tau\right)d\tau\ , \qquad (2)$$

In the type (2), as is $u\left(\tau\right)$ the excitation signal, $y\left(t\right)$ the measurement signal, $K\left(t,\tau\right)$ the general linear systems operator.

The wavelet theory is developed by the time-frequency analysis, so we can obtain:

Theorem 1: [1] Suppose $\Psi\left(x\right)$ and $\varphi\left(x\right)$ both are R wavelet and their scaling functions. According to that, we can get: $\psi\left(2^j x-k\right)$, j, $k\in Z$ composes of $L^2\left(R\right)$ complete orthogonal basis, So dose $\psi^{(j)}(j=1,2,3)$ of $L^2\left(R^2\right)$.

Thus, $K\left(t,\tau\right)$ can decompose of the following two-dimensional wavelet:

$$K(t,\tau)=\sum_j K^{(j)}(t,\tau)=\sum_j\sum_{k_1,k_2} w_{k_1,k_2}^{(j)}\psi^{(j)}(t,\tau).$$

Also, there is $\{v_{m,n}^{(j)}\}$（$j=1,2,3$） corresponding to its wavelet $\psi^{(j)}(j=1,2,3)$ expressed as

$$\psi^{(j)}(t,\tau)=\frac{1}{4}\sum_{l_1,l_2} v_{m,n}^{(j)}\psi^{(j)}(2t-l_1,2\tau-l_2).$$

Thus, $K^{(j)}(t,\tau)=\frac{1}{4}\sum_{m,n} q_{m,n}^{(j)}\psi^{(j)}(2t-m,2\tau-n)$, $q_{m,n}=w^{(j)}{}_{k_1,k_2}v^{(j)}{}_{l_1,l_2}$. (3)

Formula(3) is translated by Fouier:

$$K^j(x)=Q^{(j)}(\xi,\eta)\varphi(\xi,\eta) \qquad Q^{(j)}(\xi,\eta)=\frac{1}{4}\sum_{m,n} q_{m,n}^{(j)}e^{-i(m\xi+n\eta)}.$$

As can be seen, the problem of the time-varying system identification turns to $q_{m,n}^{(j)}$ structure.

4 the Construction of Binary Compactly Supported Orthogonal Wavelet Filter

Suppose $\{p_{m,n}\}$ supports for $\Omega=[0,2M-1]\times[0,2N-1]$, if $P(\xi,\eta)$ satisfies (3),(4),(5), so $\{p_{m,n}\}$ is the binary compactly supported orthogonal wavelet low-pass filter.

$$P(\xi,\eta)+P(\xi+\pi,\eta)+P(\xi,\pi+\eta)+P(\pi+\xi,\pi+\eta)=1 \qquad (4)$$

$$P(\xi,\eta)=\frac{1}{4}\sum_{m,n} p_{m,n}e^{-i(m\xi+n\eta)}$$

$$|P(0,0)|^2=1,|P(\pi,0)|^2=0,|P(\pi,0)|^2=0,|P(\pi,\pi)|^2=0. \qquad (5)$$

According to the Literature [2], $\{q_{m,n}^{(j)}\}$ is called orthogonal high-pass filter on the certain conditions, but also derived from $\{p_{m,n}\}$. We can use the construction of the Literature [3].

Theorem 2

Select non-negative integers $\{a_{2m'+1,0}\},\{b_{0,2n'+1}\},\{c_{2m'+1,2n'+1}\},\{d_{2m'+1,2n'+1}\}$, and make

$$\sum_{m'=0}^{M-1} a_{2m'+1,0}+\sum_{n'=0}^{N-1} b_{0,2n'+1}+\sum_{m'=0}^{M-1}\sum_{n'=0}^{N-1} c_{2m'+1,2n'+1}+\sum_{m'=0}^{M-1}\sum_{n'=0}^{N-1} d_{2m'+1,2n'+1}=\frac{3}{4} \text{ and } \sum_{m'=0}^{M-1} a_{2m'+1,0}=\frac{1}{4}$$ be

established , furthermore $(m'=0,1....,M;n'=0,1...,N)$.

$Q(\xi,\eta)$ as follows:

$$Q(\xi,\eta)=\frac{1}{4}+\sum_{m'=0}^{M-1}a_{2m'+1,0}\cos(2m'+1)\xi$$

$$+\sum_{m'=0}^{N-1}b_{0,2n'+1}\cos(2n'+1)\eta+\sum_{m'=0}^{M-1}\sum_{n'=0}^{N-1}c_{2m'+1,2n'+1}\cos[(2m'+1)\xi+(2n'+1)\eta]$$

$$+\sum_{m'=0}^{M-1}\sum_{n'=0}^{N-1}d_{2m'+1,2n'+1}\cos[(2m'+1)\xi-(2n'+1)\eta]\geq 0$$

So , $Q(\xi,\eta)$ satisfies:

$$\begin{cases}Q(0,0)=1\\ Q(\xi,\eta)+Q(\xi+\pi,\eta)+Q(\xi,\pi+\eta)+Q(\pi+\xi,\pi+\eta)=1\end{cases}$$

With regard to $P(\xi,\eta)=\frac{1}{4}\sum_{m,n}e^{-i(m\xi+n\eta)}$, suppose $|P(\xi,\eta)|^2=Q(\xi,\eta)$ [4] , and the result of derived sequence { $p_{m,n}$ } is an orthogonal low-pass wavelet filter.

{ $p_{m,n}$ } is obtained by $\sum_{m,n}^{2M-1,2N-1}P_{m,n}^2=4$ and

$$\begin{cases}\sum_{n=0}^{2N-1}\sum_{j=0}^{2M-1-m_0}p_{j+m,n}p_{j,n}=8a_{m_0,0}\\ \sum_{n=0}^{2N-1}\sum_{j=0}^{2M-1-2m_0}p_{j+2m,n}p_{j,n}=0\\ \sum_{m=0}^{2M-1}\sum_{j=0}^{2N-1-2n_0}p_{m,j+n_0}p_{m,j}=8b_{0,b_0}\\ \sum_{m=0}^{2M-1}\sum_{j=0}^{2N-1-n_0}p_{m,j+2n_0}p_{m,j}=0\\ \sum_{n'=0}^{2N-1-n_0}\sum_{j=0}^{2M-1-2m_0}p_{j+m_0,n'+2n_0}p_{j,n'}=8c_{m_0,n_0}\\ \sum_{n'=0}^{2N-1-2n_0}\sum_{j=0}^{2M-1-2m_0}p_{j+2m_0,n'+2n_0}p_{j,n'}=0\\ \sum_{n'=0}^{2N-1-n_0}\sum_{j=0}^{2M-1-m_0}p_{j+m_0,n}p_{j,n+n_0}=8d_{m_0,n_0}\\ \sum_{n=0}^{2N-1-2n_0}\sum_{j=0}^{2M-1-2m_0}p_{j+2m_0,n}p_{j,n+2n_0}=0\end{cases}\qquad(6)$$

including $(m_0=1,2...,2M-1;n_0=1,2...2N-1)$. It can be constructed the basic wavelet into (1) which gets the signal $\Delta y(t)$.

In practice, $\Delta y(t)$ depends on the choice of $\psi^{(j)}(t,\tau)$ towards $y(t)$ good or bad . If $\psi^{(j)}(t,\tau)$ makes $\Delta y(t)$ minimum in $\| y(t)-\Delta y(t)\|_2$, they are optimal. And then it is the best approximate of signal $y(t)$ to $\Delta y(t)$, however, $\| y(t)-\Delta y(t)\|_2$ is a cost function between them in fact . For a decomposition of a limited wavelet, that can calculate $y(t)$ in the sub-space coefficient. Then the cost function calculates each layer's coefficient of cost function values.

It calculates all child nodes of the cost function space according to the above definition of cost function, where is the top mark named parent node that starts from the lowest sub-node. If the cost function of the parent node is lower than the marked sub-nodes, so mark the parent one. Otherwise not marked, and pass the youngest child one to the parent., and so on, until the top. Deleted all marked child nodes, there the best wavelet basis is from the top. All sub-optimal space-based cost functions is ordered from small to large. The smaller is cost function on the best base subspace, the better is energy value as the feature classification.

5 Time-varying Linear Systems Expressed by the Wavelet's Steps:

1） $K(t,\tau)$ decomposes the two-dimensional wavelet ;

2) Select a positive integer M and N ;

3) Select the non-negative integer $\{a_{2m'+1,0}\},\{b_{0,2n'+1}\},\{c_{2m'+1,2n'+1}\},\{d_{2m'+1,2n'+1}\}$;

4) In the constraint condition $\sum_{m,n}^{2M-1,2N-1} P_{m,n}^2 = 4$, we can get $\{ p_{m,n}\}$ by nonlinear equation (6);

5) Select the best basis function, and put the calculation of the signal into the best approximate $\Delta y(t)$.

6 Conclusions

In this paper, linear operator deduces wavelet-based time-varying systems described in the modeling process. The model can describe a relatively simple way of the characteristics of system's and nonlinear time-varying.It can also be applied to other models on time-varying parameter identification with appropriate modifications. So it has engineering significance and value.

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